THE DESIGN DEPARTMENT AND THE PROBLEM OF FATIGUE RELIABILITY

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INTRODUCTION

The fatigue-resistance problem of aircraft structures becomes daily of ever-increasing importance, explaining the amount of different works on the subject. However, it should be noted that fundamental research on the physical nature of the phenomenon has been considerably more developed than the efforts made in synthesizing the problem, in order that a rational use of the results may be made by the Design Department engineer.

In an article published in 1952 [2], Mr. W. Barrois, Chief Engineer in the French Service Technique de l'Aéronautique, remarks: "It becomes increasingly important to predict the fatigue resistance of an aircraft, it being necessary to have a simple set of laws available for the design and for conducting the tests. . . ."

These simple laws have been grouped, since the publication of the work, by the same author in a report published in 1962, and having for subject fatigue crack propagation [3], based on a study made in 1952 published in 1963 [1]. Numerous references to this document will be made throughout the text.

Along with this "phenomenological theory of fatigue," to the advancement of which Mr. Barrois has so much contributed in France and about the same time, the aeronautical industry has seen the development of the reliability concept. Aircraft operating conditions becoming more and more severe, the notion of a service life limited by wear and fatigue was necessarily imposed, well before the time the Design Department had at its disposal the reliability theory in a practical form for the design stage of a new aircraft.

At the present moment, we are therefore confronted with (a) a phenomenological theory of fatigue which comprises a set of results defined in the form of laws (qualitative and quantitative) having a *deterministic character*, and (b) a reliability theory which attempts to represent the phenomena of limited life (wear, fatigue) having a *random character*. Based on these two theories, it would be desirable to place at the design engineer's disposal a mathematical model allowing him to evaluate a priori the first principles of his design while taking into account the objective, in a similar manner to that currently done in the field of static load strength. This paper proposes to define such a model and to present the results already obtained in this connection.

The principle on which these two theories are based is that the deterministic character of fatigue laws can only be true as a mean, as this phenomenon is by nature extremely scattered and always random. The link thus established also allows us to assign to the statistical parameters a physical aspect predictable by certain laws and thus determine the predictable laws of probability valid at the design stage, while taking into account the different parameters regarding the definition and the result aimed at.

We will summarily recall the main results derived from the reliability theory. They reveal the necessity of defining a distribution law for the length of fatigue life. However, owing to the very definition of reliability this law must be a function of the parameters defining the environment. This environment being random by nature, we will recall the notions necessary for its definition as well as their use in the model proposed.

NECESSARY RELIABILITY NOTIONS

Reliability (or safe operation) of a component is defined as the probability of its operating under given service conditions over a given time. It can instantly be seen that the notion of reliability is closely linked to that of failure and to the manner in which it is produced. It will therefore be necessary, when speaking of fatigue, to define exactly what is meant by failure of a structure subjected to wearout fatigue.

If we suppose for the moment that this definition be acceptable, i.e., $G(t)\,dt$, the frequency function for a period of time t at the end of which failure occurs (probability that the failure occurs in $t < T < t + dt$) the distribution function of the law of reliability will be:

$$F(t) = 1 - G(t)$$

where

$$G(t) = \int_0^t g(t)\, dt$$

This equation is true for a single component but structures can be considered as an assembly of individual components having each their proper reliability.

Two main types of systems may be discerned from which all the others are derived. From analogy with electrical circuits, we can discern:

1. Series systems, which may be defined as being the arrangement in which the failure of a single component would lead to the failure of the complete structure.
2. Parallel systems, which may be defined as being the arrangement in which the failure of all the assembly components is necessary for the structure to fail.

From the fatigue point of view, we can discern the notions of "safe-life" and "fail-safe" currently used in the aeronautical field. In fact, the parallel system corresponds to the fail-safe concept, according to a remark made by M. Barrois, only in the case where the failure of a component does not result in overloading the rest of the structure and its practically instantaneous failure. The structure is so much less fail-safe when its components of equal strength are equally loaded. A recent experimental and theoretical study by Eggwertz and Linsjö [7] confirms this point of view. Given the reliability of each of the components comprising the structure, it is possible to deduce the reliability of each of the above systems.

From the definition of the series systems, we can deduce that $F_s(t)$, the probability that no failure will occur in the structure during time t, is the probability that no failure will occur in the first component at time t, in the second at time t, and so on. If, in addition, the failures are supposed to be independent, then:

$$F_s(t) = F_1(t)F_2(t) \ldots F_i(t) \ldots F_n(t)$$

where $F_i(t)$ is the reliability of the ith component at time t.

If we are dealing with two redundant parallel components, the nonfailure probability of this type of structure is the probability that at least one of the components will continue to perform at time t.

$$F_s(t) = F_1(t) + F_2(t) - F_1(t)F_2(t)$$

which can also be expressed as:

$$1 - F_s(t) = [1 - F_1(t)][1 - F_2(t)]$$

When a number of components are arranged in parallel, the failure of the structure may be produced by a partial number of elementary failures; the reliability of such a structure may be obtained by using the coefficients derived from binomial development. If F is the reliability of each component and G its failure probability, if there are n number of components in parallel, we obtain:

$$(F + G)^n = 1$$

This expression covers all the possible combinations of failure and the nonfailure cases of parallel components.

A question is thus set at the design stage: For a given reliability of the complete structure, what reliability should be assigned to each component to achieve this? In other words, how can we share the responsibility of overall reliability between each of the components? This shareout obviously comprises an arbitrary choice which locally would have some effect, as we shall see in the design of the structure. The following share rule has been proposed [4]:

If the reliability is equally shared between an n number of components, each one would have a reliability function $F_i(t) = [F_s(t)]^{1/n}$ *for the series system defining the level of reliability.*

We prefer balancing each component by using such indices as I_i

$$F_i(t) = [F_s(t)]^{w_i}$$

where

$$W_i = \frac{I_i}{\sum_{j=1}^{n} I_j}$$

For each level I_i will be such that

$$I_i = I_{\mu i}(I_{ki} + I_{fi} + I_{mi})$$

$I_{\mu i}$ is the state-of-the-art index.

If we express

$$I_i = (K_i)^{V_i}$$

with

$$K_i = \frac{Z_i K_{b_i}}{\sum_{j=1}^{n} Z_d K_{b_d}}$$

where $K_{b_i} = \frac{10 n_{b_i}}{n_{b_c}}$

n_{b_i} = number of components at i level

Z_i = failure rate $= \frac{g_i(t)}{1 - G_i(t)}$

n_{b_c} = number of components at the most complex level

$V_i = \bar{\mu}^{T_w}$

$\bar{\mu} = \mu + T_0 \Delta u$

T_0 = number of years from a given date

μ = reliability a priori of the component, taking into account the state-of-the-art at the time the draft project was drawn up.

T_w = number of years during which the component was improved from a given date.

I_{Ki} is the complexity index

$$I_{K_i} = 1 - \exp\{-K_{b_i} + 0, b\ K_{pi}\}$$

If n_{pi} is the number of redundant components at level i, $K_{pi} = 10 n_{pi}/n_{pc}$ and n_{pc} is the number of redundant components at the most complex level.

I_f is the environmental index

$$I_{f_i} = 1 - \frac{1}{f}$$

where f = stress index estimated as lying between 1 and 100. Failure taking place at 100; $f = 1$ is the level at which there is no failure.

I_m is the operating time index

$$I_{m_i} = \frac{T_{m_i}}{T_{\mu_i}}$$

where T_m = total mission time of the component

T_μ = operating time at the level considered

The indices defined above are valid whatever phenomenon intervenes to reduce life. In particular, these considerations apply to the definition of local fatigue reliability.

In the same manner as it is necessary to define exactly the rupture at failure, it will also be necessary to define, by applying the notions recalled above, a method for breaking down the structure into simple components arranged either in a parallel or a series system. Whatever the method used, before any analysis is made three essential questions must first be answered:

1. What is the mission for which a certain reliability is required?
2. For how long (in time or number of cycles) is this reliability required?
3. In which environment, under what service conditions, will the structure be operating during the mission?

PROBABILISTIC MODEL OF THE FATIGUE PHENOMENON

The preceding reasons have shown the necessity, for making use of the reliability notion, of having a distribution function available for the fatigue lives $G(t)$.

Independently of the studies regarding the physical aspect of the fatigue phenomenon, a certain number of authors have given their attention to the representation of fatigue life by means of a known distribution function. There have thus been proposed:

(a) a normal logarithmic law
(b) a distribution of extreme values
(c) Weibull's distribution
(d) a gamma distribution or Pearson's law type III

These all have one thing in common—they are asymmetrical. Studies made have been mainly based on the agreement between these laws and by experimental results gathered from bench test specimen failures.

It can readily be seen that between the problem set by fatigue reliability and the results obtained in the statistical field, a certain number of questions may be asked, which must be answered if we wish to make use of a prediction model. Among these questions we would mention the following:

1. Among all the proposed laws, which should preferably be the one chosen to represent the life distribution?
2. How can one choose a priori the value of the statistical parameters shown in a given distribution law for a defined reliability factor?
3. How can one take into account the environment and the service conditions in the parameter values?
4. How can one, for a given reliability, a defined mission, obtain an idea of the magnitude of the dimensions to choose for structural components, in a similar manner to that made for static loads?

The following paragraphs will attempt to give some answers in order to conclude on a proposed working model.

CHOICE OF MODEL PROBABILITY FUNCTIONS

It is first of all necessary to dissipate any indetermination in the choice of a distribution function for the lengths of life. One of the difficulties in making this choice is that the fatigue statistics are generally of insufficient magnitude and thus that several laws appropriately chosen may be adjusted to the experimental results. We will therefore try to use another method for establishing the life distribution law.

Examining Forsyth's works, Barrois states in Ref. 3, p. 18, "On examining fatigue fracture in sheets subjected to repeated tensile loadings, it will be seen that quite often crack propagation is intermittent, proceeding forward in major steps; each step in fact represents partial static rupture under tensile stress." This qualitative remark suggests that the crack-forming process is a Poisson process.

If we set

K = number of elementary cracks in the break at time t (an elementary crack corresponds to one step)

λ = mean number of elementary cracks per unit time

Under these conditions:

$$\text{Probability } \{k = K\} = \frac{(\lambda t)^K}{K\,!} e^{-\lambda t} = P_K(t) \tag{1}$$

It is impossible to directly test such a law as the value of K cannot be experimentally measured. We will see later how an additional assumption allows one to obtain indirect proof.

From this law we can deduce a life-distribution law by the usual reasoning. In fact, according to Eq. (1) the probability that $K = 0$ at time t (the probability that at time t no elementary crack will be initiated) is equal to $P_0(t) = e^{-\lambda t}$; that is, t_i, t_{i+1} are two instants where two successive elementary cracks are initiated, F_i, F_{i+1}. If $t_{i+1} - t$ is smaller than t, the probability that at least one elementary crack will be initiated at time interval t is $1 - e^{-\lambda t}$, i.e.,

$$\text{Probability } \{t_{i+1} - t_i \leqslant t\} = 1 - e^{-\lambda t} = G_0(t)$$

$G(t)$ is by definition the distribution law of intervals t, and its frequency function

$$\frac{d}{dt} G_0(t) = g_0(t); \qquad g_0(t)\, dt = \lambda e^{-\lambda t}\, dt$$

in which the mean is $1/\lambda$ mean time interval between the initiation of two elementary cracks.

In particular, $g_0(t)$ is the distribution law at the time the first elementary crack appears—that is, the "nucleation" time. By similar reasoning we can calculate $g_i(t), \ldots, g(t)$, giving

$$g(t)\,dt = \frac{1}{\Gamma(K)}\, e^{-\lambda t}(\lambda t)^{K-1} d(\lambda t) \tag{2}$$

As λ is by definition the mean number of elementary cracks per unit time, we can see that the parameter for this statistical law has an obvious physical significance.

Law (2) was tested on a certain number of statistical results corresponding to the number of cycles to failure of all kinds of specimens. The results are systematically good.

It remains to be demonstrated that this law shows some particular reasons for allowing us to think that it is better adapted than another to the fatigue phenomenon.

We have seen that the crack length could be considered as the sum of the length of a number K of elementary cracks. These elementary cracks are not all of the same dimensions and their lengths vary for two reasons:

(a) In the same specimen, the dimension varies from the beginning to the end of the process owing to the variation in local stress due to the finite dimensions of the specimen.

(b) For the same crack dimension already initiated, it varies from one specimen to the other in a random manner.

We will assume that the distribution of the elementary crack dimensions is exponential, i.e., with a probability

$$\text{Probability } \{\epsilon \leqslant E \leqslant \epsilon + d\epsilon\} = \frac{1}{d}\, e^{-\epsilon/d}\, d\epsilon \tag{3}$$

where ϵ is a random variable having a mean dimension d for the elementary cracks.

This law can be directly tested. But the attempt has not been made as it is possible to do this indirectly; l being the length of a crack, it is simple to deduce the distribution law for l from Eq. (3).

We have in fact seen that the crack was the sum of K independent variables $\epsilon_1, \epsilon_2, \ldots, \epsilon_k$ having as a distribution law, the expression (3), the characteristic function being

$$\Phi(t) = \frac{1}{1 - idt}$$

The characteristic function of the distribution law l will therefore be

$$\psi(t) = \frac{1}{(1 - idt)^K}$$

which is the characteristic function of a gamma law in the form of

$$f(l)\,dl = \frac{1}{d^K \Gamma(K)} l^{K-1} e^{-l/d} dl$$

or

$$f(l)\,dl = \frac{1}{\Gamma K} (l/d)^{K-1} e^{-l/d} d(l/d) \qquad (4)$$

The crack lengths are therefore distributed according to a gamma law and the mean length is

$$\bar{l} = Kd \qquad (5)$$

where d is the mean dimension of the elementary cracks and K the number of mean cracks necessary for obtaining the length.

It is simple from Eq. (4) and by similar reasoning to that adopted in deducing Eq. (2) from Eq. (1), to find the distribution law for K in relation to parameter l/d which is considered as given.

We obviously find a Poisson law in the form of

$$\text{Probability } \{k = K\} = \frac{(l/d)^K}{K\,!} e^{-l/d} \qquad (6)$$

We have therefore from this expression

$$\overline{K} = \frac{l}{d} \qquad (7)$$

If we compare Eqs. (6) and (1) we can deduce, if the assumptions made in the forming process of elementary cracks and the distribution of their dimensions are valid, that

$$\bar{l} = (\lambda d)t \qquad (8)$$

as, particularly in expression (6) we can make $l = \bar{l}$, and $K = \overline{K}$ in Eq. (5). This formula allows an indirect test of the entire assumption.

The Sud-Aviation Central Laboratory has studied the crack propagation of 50 notched specimens in AU-4G1 light alloy subjected to alternating tensile loads on the 10-ton Schenk machine. We thus obtain a curve for each specimen having the usual exponential form. By studying the mean lengths of the cracks over a number of given cycles, the points given should approximately lie on a straight line. It is to be noted that this straight line will not pass through the origin, the parameter t representing a time interval and not absolute time. Figure 1 shows the results obtained.

We have seen that parameter K is embarrassing as it is experimentally difficult to locate. We can obtain the distribution laws for the random variables t and l independently from K, which can immediately be written:

$$\text{Probability}\ \{l \leqslant L \leqslant l + dl\} = \sum_{K=1}^{\infty} e^{-\lambda t} \frac{(\lambda t)^K}{K\,!} \frac{1}{\Gamma(K)} \left(\frac{l}{d}\right)^{K-1} e^{-l/d} d(l/d)$$

N	l	l mean
25,000	1.5 2.3 1.8 2.3 0.7 5	2.26
30,000	10.8 2.2 2.8 3.7 2 1.8 7 1.2 1.2 1.9 2.3 4.9 0.3 0.7 1.6 3.4 3.7 15	3.62
35,000	2.5 3.4 1.8 2.2 6.4 7.9 0.8 3.6 6.8 3.2 3.4 4 5.6 8.8 1.7 2.8 4.8 7.5 12.2	4.70
40,000	2.6 3.1 8.9 18 4.2 1.4 3 3.2 5 9	5.84

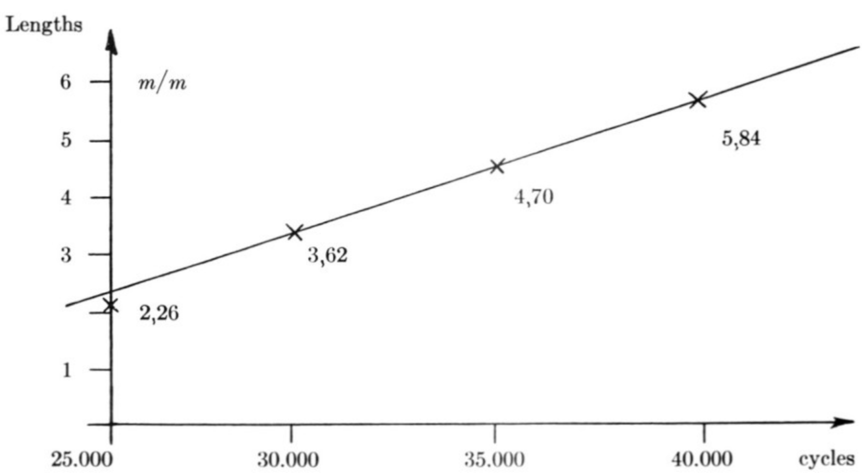

Figure 1.

$$\text{Probability } \{t \leqslant T \leqslant t + dt\} = \sum_{K=1}^{\infty} e^{-l/d} \frac{(l/d)^K}{K\,!} \frac{1}{\Gamma(K)} (\lambda t)^{K-1} e^{-\lambda t} d(\lambda t)$$

The first law gives the probability for l, from which it is relatively easy to calculate the mean; thus we find $\bar{l} = (\lambda t)\, d$. It is Eq. (8) that we obtain directly.

The second law enables us by using the statistics obtained from the Sud-Aviation Central Laboratory to test χ^2 which proves that all the assumptions made are justified and can be used as a basis for a probabilistic model of the fatigue phenomenon. The last two laws may be written thus:

$$\text{Probability } \{l \leqslant L \leqslant l + dl\} = \sqrt{\frac{\lambda t}{l/d}}\, e^{-\lambda t} e^{-l/d} I_1[2\sqrt{(l/d)\ \lambda t}]\, d(l/d) \quad (9)$$

$$\text{Probability } \{t \leqslant T \leqslant t + dt\} = \sqrt{\frac{l/d}{\lambda t}}\, e^{-l/d} e^{-\lambda t} I_2[2\sqrt{\lambda t (l/d)}]\, d(\lambda t) \quad (10)$$

by using the Bessel modified function tables to the order of 1. We have not as yet specifically identified the value l, but it should be noted that all specific values of l have the same distribution laws as l, l_R being the value of the crack length at which specimen failure occurs, l_R is a random variable having the same probability laws as l. In fact the probability that l lies between l_R and $l_R + dl_R$ is equal to

$$g(l_R)\, dl_R = \int_{l_R}^{l_R + dl_R} f(l)\, dl = [F(l)]_{l_R}^{l_R + dl_R}$$

If $f(l)\, dl$ is the distribution of l, therefore

$$g(l_R) = f(l)|_{l = l_R}$$

It is for this reason that the different laws could be tested by taking specimen time to failure, which corresponds to $l = l_R$. Specimen failure is only the demonstration of a particular value of the crack dimension. We can admit that it could be similar for a value other than that related to failure if instead of σ_R (static failure stress) we impose that the crack length does not exceed a value such that a certain stress σ_M is not attained.

PHYSICAL INTERPRETATION OF THE STATIC PARAMETERS

If we apply Eq. (5) under the above conditions we arrive at the relation

$$K = \frac{\bar{l}_R}{d}$$

This formula enables us to use a result given by W. Barrois in Ref. 3, p. 33.

If we carry out a fatigue test under normal stress σ up to failure in N number of cycles, with a crack length of l_R, the static failure stress may then be calculated as follows:

$$(\sigma_R)^*_{\text{net}} = \frac{\sigma}{1 - l_R/w}$$

where $(\sigma_R)^*_{\text{net}}$ = static failure stress

w = the specimen width

If we call out $\gamma = \sigma/(\sigma_R)^*_{\text{net}}$ the usual nondimensional ratio, the above formula enables us to express l_R in relation to γ, i.e.,

$$l_R = w(1 - \gamma) \tag{11}$$

Equations (3) and (11) give

$$K = \frac{w(1 - \gamma)}{d} \tag{12}$$

We should recall that d is the mean length of elementary cracks d and that it is also a function of γ. It is to be remarked that we can give an a priori form to the function d: $d(\gamma)$. We know, in fact, that the number of elementary cracks is zero when γ is zero, or better when γ is less than a certain value, γ_0.

The value of the numerator of Eq. (12) being finite for $\gamma = \gamma_0$, i.e., $w(1 - \gamma_0)$, it is necessary that d be infinite for $\gamma = \gamma_0$ in order that K be zero. Therefore d will be in the form of

$$d = \frac{m}{\gamma - \gamma_0} \tag{13}$$

We can justify this form by the following text quoted by Barrois (Ref. 3, p. 18): "An important point to note here is that the interval or spacing between striations increases with the applied load . . . ," and (p. 17) "The striations are in fact produced by the discontinued progress of the crack after one or more cycles." Assuming that we place ourselves at the initiation of the phenomenon, d is consequently the crack length ($K = 1$), γ being slightly greater than γ_0. Assuming that we change only the material and that we apply a constant γ (but for this material γ_0 is slightly greater), in order to initiate the first crack it is necessary to increase the load while assuming that γ remains constant (by increasing W, which has no influence on d as it does not depend on this parameter). Consequently $\gamma - \gamma_0$ decreases. As we know experimentally that d increases with the load, it is necessary that d be inversely proportional to $\gamma - \gamma_0$. Thus d thereby reveals a specific quality of the material independently of the specimen dimensions. The parameter m should have the dimension of one length and no longer depend on the stress; it assumes thus a specific value for the material from the fatigue point of view. We have, finally

$$K = \frac{w}{m}\left(1 - \gamma\right)\left(\gamma - \gamma_0\right) \tag{14}$$

This formula may easily be tested by establishing a certain number of statistics at variable stress levels by means of specimens of similar material (m, γ_0) and dimensions (W).

Values for K (K_1, K_2, K_3, K_4) are obtained by the maximum likelihood method. Parameters $M = w/m$ and γ_0 are adjusted by the method of least squares.

We have seen that $\gamma = \sigma/\sigma_R$. In the study of fatigue phenomena we usually write $\sigma = \sigma_m \pm \sigma_a$. This manner of proceeding is unsuitable for representing a function of the variable γ, the latter not being univocal.

Although of an immediately less apparent significance we prefer taking for the quantity

$$\gamma = \frac{\sigma_a}{\sigma_R - \sigma_m} \tag{15}$$

of which the absolute value is at least univocal. The results are shown in Table 1.

A second parameter remains in the distribution law: λ mean number of elementary cracks per unit time can be substituted for $\Theta = 1/\lambda$ mean initiation period of two successive elementary cracks. From Eq. (2) we have

$$K\Theta = \bar{t} = \frac{N}{f} \tag{16}$$

where $\Theta f = \bar{N}/K$. It is relatively easy to estimate $\bar{N}$ in relation to γ. We have written

$$\overline{N} = A(1 - \gamma)\, e^{B/\gamma} \tag{17}$$

Taking into account the value of K found previously, we can try to adapt a formula expressed as

$$\Theta f = C\,\frac{e^{B/\gamma}}{\gamma - \gamma_0} \tag{18}$$

The following table shows the value of the representation. The two relations, Eqs. (14) and (18), give an expression in the form of Wöhler curves for a given specimen.

TABLE 1

Reference	Specimen type and nature of load	Number of specimens	Numerical values			γ_0	M	Calculated statistically, K			Calculated by formula $K = M(1-\gamma)(\gamma-\gamma_0)$
			σ_a	σ_m	γ			$K - d$	K	$K + d$*	
Sud-Aviation	AU 4 G (light	70	573.75	701.25	0.161	0.067	229	15.32	16.83	18.34	17.5
Central Labora-	alloy)	50	956.25	1168.75	0.310	0.067	229	33.27	35.88	38.49	37.22
tory, 1961	4250 kg	50	1338.75	1636.25	0.512	0.067	229	48.15	52.	55.15	48.21
	notched 2 mm	56	1692.56	2068.68	0.775	0.067	229	33.41	35.88	38.35	35.36
	thick 15 mm										
	width 68 mm										
	alternating load										
EFA-R : 59	Nuts and bolts										
Static strength	0.15	100	250	1000	0.192	0.076	58.3	12	13.1	14.2	12.57
and fatigue	0.										
properties of	0.5	99	400	1000	0.308	0.076	58.3	15.44	16.7	17.97	15.44
threaded bolts,	0.04										
W. Weibull, 1955	0.04	99	600	1000	0.462	0.076	58.3	13.78	15.32	16.86	16.84
	57 kg/mm	99	800	1000	0.616	0.076	58.3	12.66	14.05	15.44	15.44
	8 mm										
	2297.7 kg										
	alternating load										
EFA-R 68	Aluminum alloy	74	18	18	0.368	0.355	77.23	0.36	0.65	0.94	0.63
Static strength	75 ST										
and fatigue		44	20.5	20.5	0.442	0.355	77.23	2.60	3.46	4.32	3.74
properties of	= 66.8 kg										
unnotched circu-		44	23.5	23.5	0.542	0.355	77.23	4.94	6.09	7.24	6.61
lar 75 ST speci-	bar. = 60 mm										
mens subjected		74	26.5	26.5	0.657	0.355	77.23	7.87	8.94	10.01	8.46 8
to tensile loading,	Repeated load										
W. Weibull, 1956											

* α confidence interval 95%.

TABLE 2

Designation	γ	$L(\gamma - \gamma_0)$	N_-	N_+	K_-	K_+	$L_-(\Theta f)$	$L_+(\Theta f)$	B_-	B_+	$L_-(C)$	$L_+(C)$
Sud-Aviation	0.161	−2.36446	303.326	363.620	15.32	18.34	9.71349	10.07473	0.67200	0.65139	3.175	3.664
= 0.067	0.310	−1.41468	35.855	41.773	33.27	38.49	6.83685	7.13528			3.254	3.619
	0.512	−0.80968	7.896	8.924	48.15	55.15	4.96403	5.22215			2.841	3.140
	0.775	−0.34532	3.085	4.269	33.41	38.35	4.38753	4.85022			3.175	3.664
W 59	0.192	−1.31678	608.400	720.440	12.	14.2	10.66536	11.00272	0.74049	0.74149	5.491	5.824
= 0.076	0.308	−0.95711	125.724	146.760	15.	17.97	8.85313	9.18853			5.491	5.824
	0.462	−0.61988	29.977	35.225	13.78	16.86	7.48325	7.84713			5.260	5.622
	0.616	−0.36816	9.334	10.924	12.66	15.44	6.40446	6.76029			4.834	5.188
W 68	0.368	−4.34282	909.160	3869.160	0.36	0.94	13.78217	16.19022	1.66893	2.57001	4.904	4.863
= 0.355	0.442	−2.44185	118.023	223.003	2.60	4.32	10.21539	11.35946			3.997	3.103
	0.542	−1.67665	39.564	59.772	4.94	7.24	8.60604	9.40092			3.850	2.982
	0.657	−1.19732	22.897	29.003	7.87	10.01	7.73518	8.21209			3.997	3.103

Designation	γ	$a = L(c) - L(\gamma - \gamma_0)$	$L_-(\Theta f)$	$L_+(\Theta f)$	B/γ	$a + B/\gamma$
Sud-Aviation	0.161	5.56446	9.71349	10.07473	4.16149	9.72595
$L(C) = 3.2$	0.310	4.61468	6.83685	7.13528	2.16129	6.77597
$B = 0.67$	0.512	4.00968	4.96403	5.22219	1.30859	5.31827
	0.775	3.54532	4.38751	4.84969	0.86451	4.40983
W 68	0.368	8.14282	13.78217	16.19022	5.64945	13.79227
$L(C) = 3.8$	0.442	6.24185	10.21539	11.35946	4.70361	10.94546
$B = 2$	0.542	5.47665	8.60604	9.40092	3.83579	9.31244
	0.657	4.99732	7.73518	8.21209	3.21329	8.21061
W 59	0.192	6.51678	10.66536	11.02272	4.16666	10.8344
$L(C) = 5.2$	0.308	6.15711	8.85313	9.18853	2.59740	8.75451
$B = 0.8$	0.462	6.81988	7.48325	7.84713	1.73160	7.55148
	0.616	5.56816	6.40446	6.76029	1.29870	6.86686

We have seen that the life-distribution functions are obtained by the function $I(u, p)$ from Pearson's tables, in which

$$p = K - 1; \qquad \sqrt{p + 1}\,\mu = \lambda t$$

where

$$N = ft = f\Theta\sqrt{p + 1}\,\mu = f\Theta\sqrt{K}\,\mu$$

thus

$$N = D\left(\frac{1 - \gamma}{\gamma - \gamma_0}\right)^{1/2} e^{B/\gamma}\mu \tag{19}$$

μ is determined for a given probability.

Parameters K and λ are therefore defined in relation to the specimen dimension W, to the magnitude of parameter γ representing the stress state, and to the three constants of which the value depends on the material chosen and can be determined by preliminary tests on any type of specimen.

We wish to make a remark regarding the formula for calculating K allowing extrapolation of the results derived from a specimen, to those of a structure.

The life of a structure is a function, not of the failure of the structure, which should never happen, but of the crack length produced on a given dimension. In the case of a structure $l_R = l_M$ will be maximum length of a crack not to be exceeded for dimension W, which is determined by stress $\sigma_R = \sigma_M$ and should not be exceeded while taking into account the safety factors defined elsewhere, and corresponding to a fictitious failure. In other words, it will be sufficient to consider that failure of the specimen occurs prohibiting the continuation of crack propagation. This prohibition, instead of being physical, can simply be according to the rules without deteriorating the fundamental phenomenon. In the estimation of γ, we can substitute σ_M for σ_R.

It should also be remarked that we can pass from the variable stress with constant parameters (σ_m, σ_a) to the random stress arising, for instance, from atmospheric turbulence. If we know the distribution for γ it is simple to deduce by use of Eqs. (14) and (18) the distribution for K and λ, i.e., $f_K\,dK$ and $f_\lambda\,d\lambda$; these distributions expressed by means of the two parameters m_γ, σ_γ are represented in the distribution for γ; the life distributions, for example, would be expressed in relation to these two parameters by the expression

$$h(t)\,dt = \left[\iint g(t; K, \lambda)\, f_K f_\lambda\, dK\, d\lambda\right] dt \tag{20}$$

This is true only for the stationary process—i.e., with σ_m constant and σ_a randomly variable.

It should be noted that the form given to γ enables its distribution to be readily obtained from σ_a; one and the other only differ in this case by a constant factor equal to

$$\frac{1}{\sigma_M - \sigma_m}$$

It is therefore the distribution of σ_a which will govern the overall calculation, enabling l_M to be determined and consequently W for a reliability given a priori.

RANDOM STRESS

The random nature of stress is the consequence of variations in steady permanent flight imposed by random phenomena—atmospheric turbulence, flight maneuvers, jet noise, landing shocks, taxiing, etc. Since clear-weather turbulence has been encountered at high altitudes which can subject the aircraft to random vibration conditions during one or two hours at 0.9 Mach, this reason has become of relative importance. It may be retained as an example. Studies made by H. Press have shown that atmospheric turbulence may be considered locally as a steady and gaussian random phenomenon; the σ standard deviation of one of the speed components varying from one point to another. A further assumption appears to be valid—the statistical characteristics of the turbulence in relation to an aircraft crossing the region are not variable in time to any appreciable extent. In other words, the turbulence pattern may be "frozen" in space.

The aircraft constitutes a linear aperiodic filter having a high time constant. The developed stresses resulting from the random phenomena of a Laplace nature will have themselves, a fortiori, this nature.

The fatigue phenomenon is characterized by parameter γ which we have written as equal to $\sigma_a/(\sigma_M - \sigma_m)$. In this parameter σ_a represents the absolute value of the maximum variable stress. In the present case, it is considered as a Laplace random variable, of which it is agreed that the distribution law of the maximum values should be sought.

We know the work that S. O. Rice has done on noise analysis which in this case is entirely transposable.

Assuming that the autocorrelation function is known $\psi(\tau)$ of σ_a. We can show that a sufficiently approximate expression of the frequency function of maximum values is

$$\frac{1}{2\pi\psi_0}\left[-\frac{\psi_0''}{\psi_0}\right]^{1/2}\sigma_a e^{-\sigma_a{}^2/2\psi_0}d\sigma_a\,dt \qquad (21)$$

$R = \sigma_M - \sigma_m$ being a constant coefficient, the frequency function $p(\gamma)d\gamma$ in unit time can be written as:

$$p(\gamma)d\gamma = \frac{R^2}{2\pi\psi_0}\left[-\frac{\psi_0''}{\psi_0} \right]^{1/2} \gamma e^{-(R^2/2\psi_0)\gamma^2}\, d\gamma \qquad (22)$$

The use of this frequency function implies the knowledge of $\psi(\tau)$ for a particular point of the aircraft, knowing the initial random phenomenon producing the random stress.

This function may be obtained by several methods. The current theories, whether using a gust unit, or an impulse or spectral component, make use of the aircraft transfer function for calculating the applied loads. This type of formula is known and has been the subject of numerous works. B. Etkin in particular has summarized this in Ref. 5.

However, in the draft project stage and as a first evaluation it seems preferable to make use of a more simple method. The design office knows how to calculate the dynamic response of an aircraft under sinusoidal loads. This calculation can be extended to a random load by replacing the periodic function by a pseudo-random function.

The method that comes to mind when we speak of periodic functions is the use of harmonic analysis. However, by doing this we find the Fourier-Stieltjes integral, which is valid only for damped phenomena in time and high temporal correlation. We would obtain a correct representation of turbulence by using the Fourier distribution transforms in the L. Schwartz sense. It would again be necessary to know the types of distribution to be used. The functions that M. J. Bass has established and designated *pseudo-random* seem to be easier to handle.

The problem set by M. Bass is as follows [5]. Given a direct image of the functions $f(t)$ having the following properties: It is the functions of time that represent a large-scale infinite and permanent phenomenon. As they are very irregular in detail, we could think of representing them by discontinuous functions. In practice, it is nearly always best to deal with $f(t)$ as a continuous and differentiable function. This function will always be characterized by its mean properties. The correlation function of $f(t)$ at the two instants t and $t + h$ must be a continuous function of h which damps out very quickly as h increases without giving rise to any irregular oscillations. In Ref. 6 will be found an application of the equations to the partial derivatives giving a turbulent solution to the heat equation and Burger's equation.

Based on a representation of atmospheric turbulence by a pseudo-random function, we can resolve the flight equations and obtain a pseudo-random solution representing the random variations of the applied load. It thus constitutes to some extent a global Monte Carlo method.

CONCLUSIONS

On condition that some experimentation is carried out, for a given material, to determine three independent constants for the structure, the model presented can be used to determine beforehand the dimensions required for obtaining the given reliability of a particular structure.

It is understood that even if the calculations of material strength are checked by strength tests of partial or complete structures before putting them into service, it will be necessary to check by tests the fatigue behavior of the components thus calculated.

Owing to the progressive nature of the cracks, it seems that very frequent inspection during the initial service period of the first aircraft would enable the gathering of statistical data in sufficient quantity to test the laws defined above.

An initial evaluation will be obtained by tests of the type similar to those carried out on the Caravelle. But valid knowledge cannot be obtained before an inspection in the course of time, and sufficiently close, to allow a test, in the mathematically statistical sense, of the distribution laws adopted.

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COMMENTARY

S. EGGWERTZ (*The Aeronautical Research Institute of Sweden, Bromma, Sweden*): M. Eugène has referred to a recent report by Lindsjö and myself [7]. He claims that our report would confirm the view that a structure consisting of a number of parallel members is not much of a fail-safe structure, if the failure of one member results in a considerable rise of the loading on the remaining members, which is usually the case.

This is not quite true—at least it is not the full story. Our testing, which is only very preliminary, was carried out with constant amplitudes, and the members were *not* inspected at regular intervals. They were run to complete failure, without any repairs or replacements being made during the test procedure. According to my view, a structure can never be fail-safe without inspections or an automatic warning system. Our test specimen with six parallel members could consequently not act as a fail-safe structure.

In our theoretical study, on the other hand, we treated the more general case with regular inspections at intervals considerably shorter than the crack propagation time. The loading was assumed to be an exponential gust load spectrum. The study seems to indicate rather clearly that it is quite feasible to make a structure consisting of parallel elements, to behave as a fail-safe structure, and to obtain a sufficiently low probability of failure, even under the conditions mentioned by M. Eugène.

(No reply required.)